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Turbulence Structure in the Axisymmetric Free Mixing Layer

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The turbulence structure of the mixing layer of a 12.7-cm axisymmetric air jet discharging into a large room has been experimentally investigated at an exit speed of 30 m/s. Streamwise evolutions of the profiles of mean velocities U and V; the fluctuation intensities u', v', and w'; and the Reynolds stress $\overline{uv}$ have been documented for the two asymptotic initial conditions: laminar and fully developed turbulent boundary layers. Even though these profiles achieve similarity well before the end of the potential core, the evolutions of the mixing layer width and peak values of u', v', w', and $\overline{uv}$ show that the effects of the initial condition continue to persist until the end of the layer ($Re_x \approx 10^6$). The transverse separations between the peaks of u', v', $\overline{uv}$ and the mixing layer center have been discussed on the basis of their spatial distributions over the extent of the 'preferred mode' coherent structures educed in an axisymmetric mixing layer.

Nomenclature

B	= shear layer width
D	= jet diameter
U,V	= streamwise and radial mean velocities
U^+,y^+	= universal coordinates for a turbulent boundary layer
u',v',w'	= rms of streamwise, radial, and azimuthal velocity fluctuations
$\overline{uv}$	= Reynolds stress
x,y	= steamwise and radial coordinates
x_0	= virtual origin
y_n	= y location at which $U/U_e = n$
δ^*,θ	= displacement and momentum thicknesses
δ_ω	= vorticity thickness $\equiv U_e/(\partial U/\partial y)_{\max}$
ω	= mean vorticity

Superscripts

$(\bar{\ })$	= time-average
$(\ )'$	= rms

Subscripts

p	= peak value
e	= exit value
PL	= peak value for initially laminar case
PT	= peak value for initially turbulent case
$y_{(\ )_p}$	= y location for the peak value of ( )

Introduction

THE turbulence structure in an incompressible free shear layer is a function of the boundary condition, the initial condition, and the Reynolds number. According to the concept of asymptotic invariance, the free shear layer should be independent of the Reynolds number provided it is large enough. From the concept of the local invariance, the streamwise variation of the flow time scale being slow, the flow should acquire dynamic equilibrium or self-preservation with increasing x. Accumulated data on mixing layers, however, reveal large discrepancies between results obtained in different investigations,[1-8] including in the simple measures like spread rate, virtual origin, or asymptotic peak turbulence intensity. (For a review of the discrepancies see Ref. 8.) Even though these discrepancies are most probably related to large-scale coherent structures and their interactions,[1,9-20] continuing advances in the understanding of the coherent structure dynamics have not yet helped explain these differences; rather, these studies have raised more unanswered questions. In fact, it is not likely that these discrepancies in time-average measures can be quantitatively rationalized through advances in the understanding of coherent structures. Thus, along with studies of coherent structures, it is still necessary to continue studies of time-average features of the mixing layer, the latter also being important in technological applications and testing of prediction codes.

Studies showing strong dependence of the near-field of a plane jet on the exit flow characteristics[21] suggested that these discrepancies may be related to the initial condition,[5,6,22] this speculation having been confirmed by data in a small circular jet.[8] However, data on spread rate and peak turbulence intensity u_p'/U_e in some recent investigations[7,10,23] suggest that the plane mixing layer is independent of the initial condition; this appears to occur for $Re_x \geq 5\times10^5$. Pending the completion of a large plane mixing layer facility,[24] the near-field of a readily available 12.7-cm air jet was investigated[25] in order to test the previously mentioned claim. In the self-preserving region, it was found that for two distinctly different initial conditions (i.e., laminar and fully developed turbulent exit boundary layers), even though the peak turbulence intensity was independent of the initial condition, the spread rate remained distinctly different for the two cases. It was also found that for either initial condition, the self-preserving measures of the mixing layer were independent of whether the jet nozzle had a thin lip or the jet discharged through a large end plate.

Since the completion of earlier work,[25] which was based on longitudinal mean and turbulent velocity data only, the authors became interested in further details of the flow, but could not locate data on the radial mean and fluctuating velocities, azimuthal fluctuating velocity, and Reynolds stress in the axisymmetric mixing layer. Hence the motivation for the present study. Since the single-wire measurements of the streamwise mean and fluctuating velocities in the authors' earlier work[25] showed no noticeable effect of the end plate on the average measures of the axisymmetric mixing layer, it was decided that data will not be taken with a large end plate. Thus, an X-wire was used to measure the evolutions of the profiles of $V(y), v'(y), w'(y)$, and $\overline{uv}(y)$ in the same mixing layer reported in Ref. 25. These additional data in this paper, thus, complete the documentation of the time-average measures of the turbulence structure of the axisymmetric mixing layer. Measurements in the axisymmetric mixing layer having been extremely scant, these data are likely to be of technological significance and also serve as references for

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calibration/development of turbulent shear flow theories/codes.

Procedure

The experiments were carried out in a circular air jet facility where the jet emerges through a 12.7-cm nozzle with a 36:1 contraction ratio and has a 20-deg bevelled lip at the exit. The details of the apparatus are given in Ref. 25. The exit plane centerline is the origin of the coordinates; x is aligned with the jet axis increasing downstream, and y increases radially. The two initial conditions, i.e., laminar and fully developed turbulent boundary layers, were documented with single-wire measurement at 0.5-mm downstream from the lip. For the initially laminar case, the mean velocity profile with a shape factor value of 2.54 agreed well with the Blasius profile. The mean velocity profile in the turbulent case, when plotted in the universal coordinates (U^+, y^+), agreed well with that of the (flat plate) equilibrium boundary layer; the friction velocity was deduced from the measured mean velocity profile through the cross-plot technique; the wake strength also corresponded to that for the flat plate boundary layer. The longitudinal fluctuation intensity profiles for the two cases were similar to those expected of the two boundary layers. (For further details of the documentation of the initial condition, see Ref. 25.)

The measurements were made with standard linearized constant temperature anemometer (TSI-1050) systems. The standard end flow X-probe (TSI-1241), with 3.8-μm welded tungsten wires, was aligned with the jet centerline at the exit by following the yawing calibration technique.[26] Data acquisition at different radial positions was done without changing the inclination of the probe axis, and probe traverse was performed under remote computer control (HP 2100S). At each measurement location, the well-matched linearized X-wire signals were directly digitized and the statistics computed digitally. Since the characteristic time scale in the mixing layer increased linearly with x, the integration time was accordingly increased at successive downstream stations; without this, data scatter was found to increase progressively with increasing x.

Results and Discussion

Data presented here correspond to the same flow conditions as in Ref. 25 (i.e., exit mean velocity $U_e = 30$ m/s, and exit centerline longitudinal fluctuation intensity $u_e'/U_e = 0.3\%$). Unlike the plane mixing layer, the axisymmetric layer has an imposed length scale, namely, the jet diameter D. While sufficiently close to the exit, i.e., for $\theta/D \ll 1$, the axisymmetric layer is similar to the plane layer, the similarity should disappear with increasing x, perhaps for $x/D \gtrsim 1$. Thus, in order to characterize the entire axisymmetric mixing layer, an intrinsic length scale such as its local thickness should be used as the appropriate length scale. The thickness can be represented by different measures, e.g., the width

$$B \equiv y_{0.95} - y_{0.1}$$

or the momentum thickness

$$\theta \equiv \int_{-\infty}^{\infty} (U/U_e)(1 - U/U_e)\,\mathrm{d}y$$

or the vorticity thickness

$$\delta_\omega \equiv U_e/(\partial U/\partial y)_{\max} = (1/|\omega|_{\max}) \int_{-\infty}^{\infty} |\omega|\,\mathrm{d}y$$

$y_{0.95}$ and $y_{0.1}$ are the transverse locations across the mixing layer, where U/U_e is 0.95 and 0.1 respectively. A number of authors have used the downstream distance x from the exit or from the virtual origin (i.e., $x = x_0$) as the characteristic length scale. In the self-preserving region, any measure of the shear layer width should be proportional to $x - x_0$. Due to the uncertainty involved in the determination of x_0, a length scale obtained from the local velocity profile seems to be a preferred choice. Liepmann and Laufer[3] used θ as the length scale.

In Ref. 25, we used

$$\theta_{0.1} \equiv \int_{y_{0.1}}^{\infty} (U/U_e)(1 - U/U_e)\,\mathrm{d}y$$

as the length scale; the integration was terminated at $y = y_{0.1}$ in order to avoid the hot-wire measurement inaccuracies on the zero-speed side due to the entrainment velocity, high-turbulence intensity, and flow reversal.[19,27,28] Ideally, δ_ω is a better choice of the length scale as it does not involve the low-speed side data; in this paper, we will use δ_ω as the characteristic length scale. The uncertainty in the determination of δ_ω, typically introduced in the direct differentiation of the measured velocity profile $U(y)$, has been significantly reduced by obtaining the value of δ_ω from the slope at $U/U_e = 0.5$ of the error function profile least squares-fitted to all $U(y)$ data for $y_{0.95} < y < y_{0.1}$. Note that the potential improvement in δ_ω by using a smaller $U(y)$ segment around $y = y_{0.5}$ would be typically offset by the increasing influences of data uncertainty. δ_ω obtained from $U(y)$ data for the ranges $y_{0.95} < y < y_{0.1}$ and $y_{0.75} < y < y_{0.3}$ were within the data scatter. Our data as well as those of Brown and Roshko[1] suggest that $\delta_\omega \approx 5\theta$.

Figures 1a-d show the streamwise evolutions of the profiles of longitudinal mean velocity $U(y)$, transverse and azimuthal fluctuation intensities $v'(y)$, and $w'(y)$, and the Reynolds stress $\overline{uv}(y)$ for the initially laminar case. The corresponding evolutions for the initially turbulent case are shown in Figs. 2a-d. $U(y)$ and $u'(y)$ profiles measured with the single wire were already presented as functions of $(y_{0.5} - y)/\theta_{0.1}$ in Ref. 25. Self-preserving region profiles of $U(y)$ and $u'(y)$ obtained with the X-wire agreed identically with those obtained with the single wire. So, only $U(y)$ profiles from the X-wire data when the wires are in the (x,y) plane are presented here for transverse coordinate reference. The profiles of $U(y)$, $v'(y)$, $w'(y)$, and $\overline{uv}(y)$ in physical coordinates in Figs. 1 and 2 show the effects of the initial condition vividly.

For both initial conditions, the mean velocity profile achieves similarity much earlier in x than the profiles of the fluctuation intensities; the Reynolds stress profiles take the longest to achieve similarity. It should be noted that the peaks of v', w', and $\overline{uv}$ for the initially laminar case rapidly increase with x near the exit to maximum values, and then rapidly decrease before dacaying slowly to the respective self-preserving values. For the initially turbulent case, the overshoot and return to constant values are gradual, and the extent of overshoot is much smaller. These kinds of trends have been observed in previous investigations in plane and axisymmetric mixing layers.[5-8,15] The large peaks near the lip in the initially laminar case are associated with instability, roll-up, and interactions like pairing [6,14,15,18] preceding the breakdown process. Even though the initially fully turbulent shear layer also forms into large-scale coherent vortical structures,[19,20] the roll-up process appears less energetic than that in the laminar case. Note that as the end of the potential core is approached, the spreading of the fluctuation intensity profiles includes potential flow fluctuations both inside and outside the mixing layer. These potential fluctuations also contribute to fluctuation intensities u', v', and w', but there should be no associated Reynolds stress; Figs. 1d and 2d do show that $\overline{uv}(y)$ profiles are narrower than the corresponding $u'(y)$, $v'(y)$, and $w'(y)$ profiles. Furthermore, the spreading of the intensity profiles is larger on the core side than on the outer side. This is to be expected because the core potential fluid is exposed to the "massaging" effect of

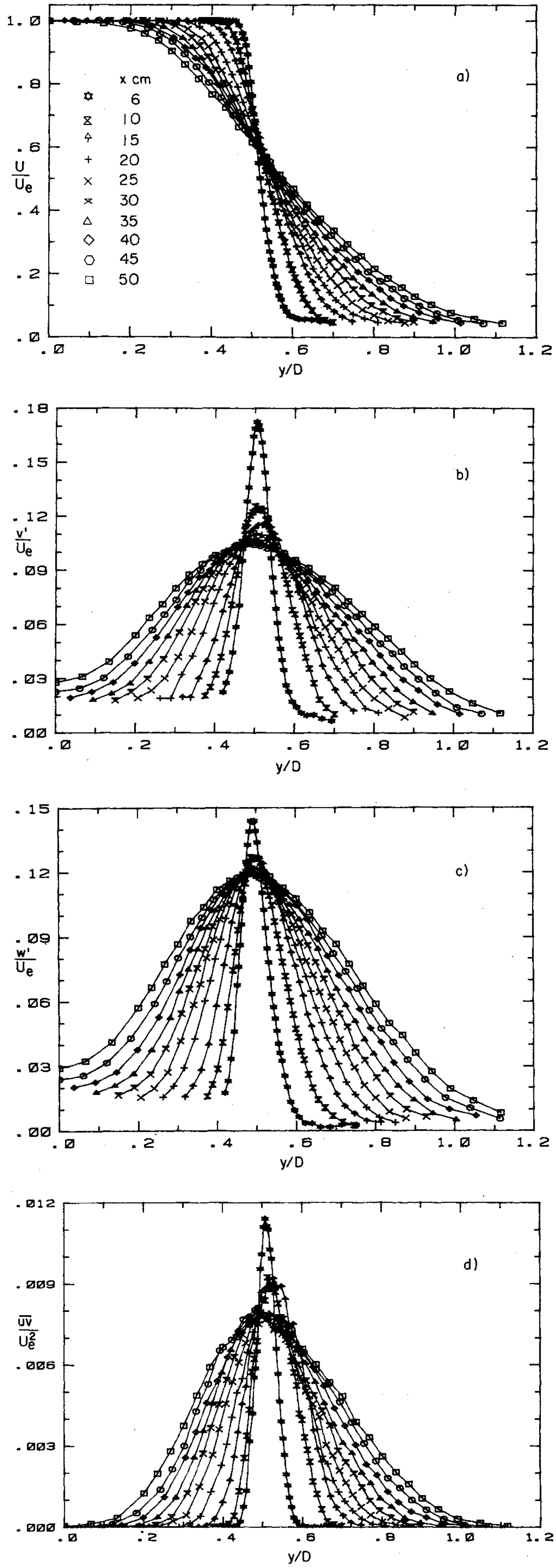


Fig. 1 Streamwise evolutions of the profiles for the initially laminar case: a) mean velocity $U(y)$; b) transverse fluctuation intensity $v'(y)$; c) azimuthal fluctuation intensity $w'(y)$; d) the Reynolds stress $\overline{uv}(y)$.

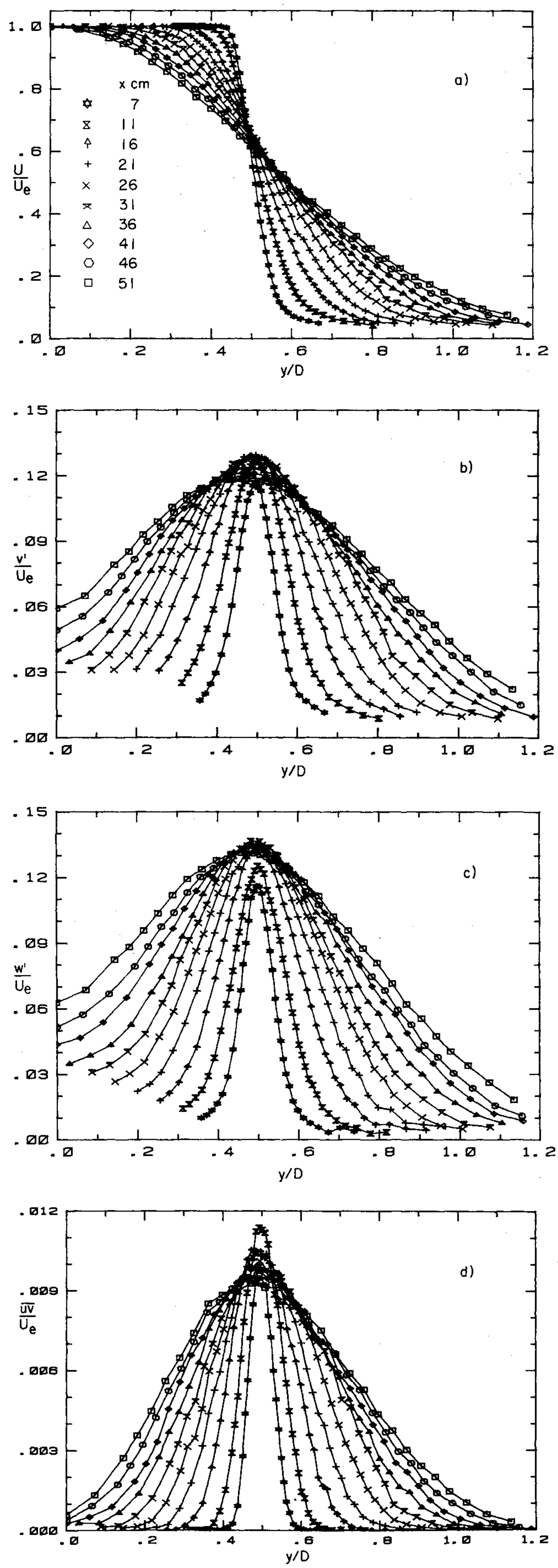


Fig. 2 Streamwise evolutions of the profiles for the initially turbulent case: a) mean velocity $U(y)$; b) transverse fluctuation intensity $v'(y)$; c) azimuthal fluctuation intensity $w'(y)$; d) the Reynolds stress $\overline{uv}(y)$.

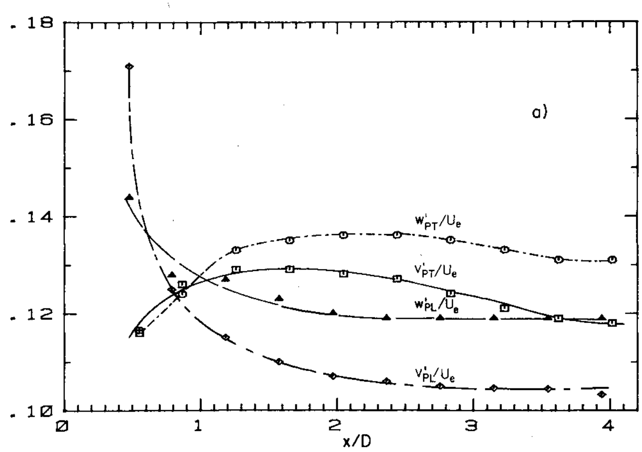


Fig. 3 a) Streamwise evolutions of v'_{PL}, w'_{PL}, v'_{PT}, and w'_{PT}.

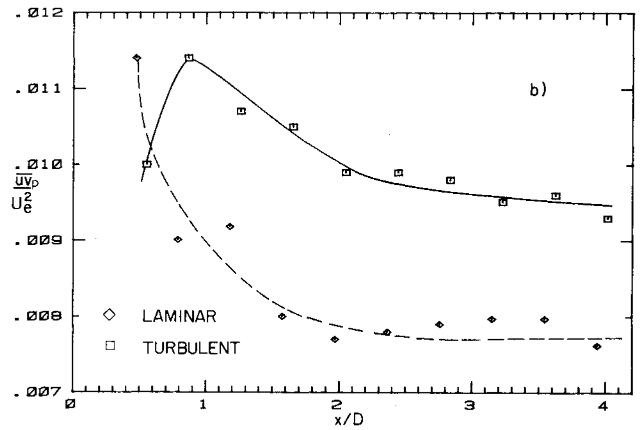


Fig. 3 b) Streamwise evolutions of the peak Reynolds stress $\overline{uv}_P$.

motions in the mixing layer all around. Consequently, the "footprint" of the large-scale eddy motions in the mixing layer is much stronger on the jet centerline side than on the outer side of the layer. Not only the mean velocity profiles but also the turbulence intensity and Reynolds stress profiles spread faster in the initially turbulent case than in the initially laminar case. The early presence of broadband turbulence in the former case presumably hastens spectral broadening,[25] and thus produces more rapid spread.

The streamwise variations of the peak values v'_p and w'_p of the fluctuation intensity profiles are shown in Fig. 3a, and the variation of the peak value $\overline{uv}_p$ of Reynolds stress profiles is shown in Fig. 3b. The second subscript (i.e., L or T) identifies the initially laminar or turbulent case. In each case, the peak value and its transverse location were obtained from a normal distribution curve least-squares-fitted through 11 data points around the measured peak value (i.e., near the center of the mixing layer). Since detailed profiles of X-wire data in the axisymmetric mixing layer have not been reported so far, prior plane mixing layer data will be used for comparison whenever appropriate.

For the initially laminar case, v'_{pL}, w'_{pL}, and $\overline{uv}_{pL}$ achieve asymptotic peak values at $x/D \approx 2$. $u'_p(x)$ data (shown in Ref. 25) are also consistent with this observation. Note that in the initially turbulent case, none of the quantities v'_{PT}, w'_{pT}, and $\overline{uv}_{PT}$ have reached exactly constant values. However, their variation with x near the end of the potential core being very slow, we can take the average of the values at the last three stations to denote the asymptotic values. The peak asymptotic values of u', v', w', and $\overline{uv}$ in the present work and those in some single-stream plane mixing layers,[2-4,29] for both initial conditions, are compared in Table 1. Note that for either initial condition, the asymptotic values in the plane and axisymmetric mixing layers are comparable, in spite of the inherent differences between the geometries of the two flows. The breakdown of the coherent structure in the initially laminar axisymmetric mixing layer occurs through the evolution of strong azimuthal lobe structures.[12,13,15,28] On the other hand, the two-dimensional large-scale coherent structures in the plane mixing layer appear to survive for much longer streamwise distances; even if the breakdown is similar, the three-dimensionality does not appear to be as strong in the plane mixing layer as in the axisymmetric layer. These differences may suggest different evolutions of the plane and axisymmetric mixing layers, the differences presumably being due to different strengths of upstream feedbacks of the coherent structures. It is indeed surprising that the asymptotic values are the same for both configurations.

It should be mentioned that Bradshaw[6] and Yule[15] reported that $v'(x)$ and $w'(x)$ along the $y=D/2$ (i.e., constant radius) line reach indentical values in the self-preserving region. Since the peak values do not occur along the $y=D/2$ line (see later), we cannot use these data for comparison with ours. Note that the asymptotic peak values for the initially turbulent case are consistently higher than the corresponding values for the initially laminar case.

Figures 4a-e show the nondimensional profiles of U, V, u', v', w', and $\overline{uv}$ as functions of the transverse coordinate $(y_{0.5}-y)/\delta_\omega$ in the self-preserving region for the initially laminar case. The corresponding profiles for the initially turbulent case are shown in Figs. 5a-e. The $u'(y)$ data in Figs. 4a and 5a were obtained with the X-wire but are indistinguishable from single-wire data.[25] In these coordinates, the U profiles appear identical for both initial conditions, but $V(y)$ profiles appear noticeably different mainly because of the magnified scale used. Note the large relative scatter in V data on the low-speed side, being aggravated by high-fluctuation intensity and flow reversal.[19,27,28] It has not been possible to compare the $V(y)$ data with others' data because the authors could not find any $V(y)$ data in any mixing layer study. It is clear that all the profiles indicate achievement of similarity.

An examination of Figs. 4 and 5 indicates that the profiles are not centered with respect to each other. That is, the locations $y_{u'_p}$, $y_{v'_p}$, $y_{w'_p}$, and $y_{\overline{uv}_p}$ of the peak values of $u'(y)$, $v'(y)$, $w'(y)$, and $\overline{uv}(y)$, respectively, do not coincide, and

Table 1 Asymptotic peak values

	Initially laminar case				Initially turbulent case				Configuration
	u'/U_e	v'/U_e	w'/U_e	$\overline{uv}/U_e^2$	u'/U_e	v'/U_e	w'/U_e	$\overline{uv}/U_e^2$	
Present study	0.159	0.105	0.12	0.008	0.166	0.12	0.13	0.0095	Axisymmetric
Liepmann and Laufer[3]	0.148	0.1	–	0.008	–	–	–	–	Plane
Patel[4]	0.178	0.11	0	0.010	–	–	–	–	Plane
Wygnanski and Fiedler[29]	–	–	–	–	0.176	0.13	0.15	0.0095	Plane
Champagne et al.[2]	–	–	–	–	0.164	0.12	–	–	Plane

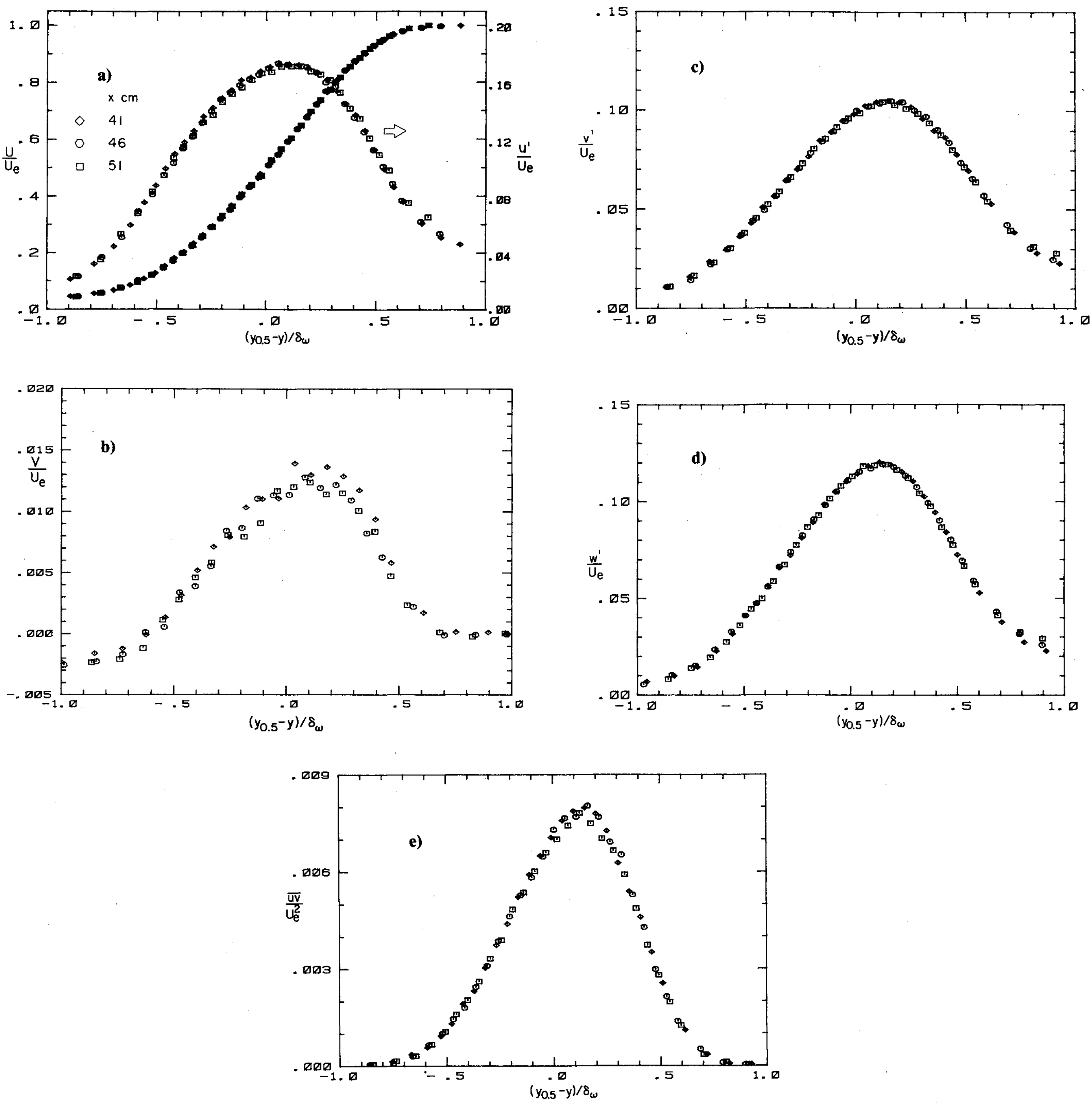


Fig. 4 Self-preserving profiles for the initially laminar case: a) $U(y)$ and $u'(y)$; b) $V(y)$; c) $v'(y)$; d) $w'(y)$; e) $\overline{uv}(y)$.

each is different from the location $y_{0.5}$ of the middle of the mixing layer. Note also that $y_{0.5}$ does not coincide with the (constant radius) axial line $y=D/2$, as is to be expected because of radially outward shift of the mixing layer owing to entrainment and viscous diffusion. The streamwise variations of the displacements $y_{u'_p}$, $y_{v'_p}$, $y_{w'_p}$, $y_{\overline{uv}_p}$, and $y_{0.5}$ are shown in Fig. 6a for the initially laminar case and in Fig. 6b for the initially turbulent case. Note that the location of each peak was obtained by least-squares fit of a normal distribution curve through 11 data points near the peak of each of $u'(y)$, $v'(y)$, $w'(y)$, and $\overline{uv}(y)$. For both initial conditions, the center of the mixing layer shifts away from the jet axis with increasing x. The peaks of u', v', w', and $\overline{uv}$ all occur on the high-speed side: $y_{v'_p}$ is farthest away from the middle of the mixing layer and $y_{u'_p}$ is the nearest; $y_{\overline{uv}_p}$ falls between $y_{u'_p}$ and $y_{v'_p}$ and is essentially on the constant radius line through the lip, i.e., the $y=D/2$ line. Note that for the two initial conditions, even though the thickness of the mixing layers is essentially the same near the end of the potential core (see Ref. 25), the separations between $y_{u'_p}$, $y_{v'_p}$, $y_{w'_p}$, $y_{\overline{uv}_p}$, and $y_{0.5}$ are distinctly different at all x until the end of the potential core.

The occurrences of the peak, e.g., $y_{u'_p}$ and $y_{\overline{uv}_p}$, noticeably away from $y_{0.5}$, are interesting because their separations are convincing reminders of the limitations of the gradient transport type model with a constant eddy viscosity. However, a general statement claiming that other transport terms, e.g., advective, turbulence, and pressure transports are significant, would add nothing to the understanding of the flow physics. In order to understand the phenomena, one must consider the coherent structures of the flow. In fact, even large-scale negative production induced in a mixing layer by controlled perturbation, first observed by the authors[30] and later by Wygnanski et al.,[31] was well explained by us in terms of the educed coherent structures and their interactions like pairing.[28]

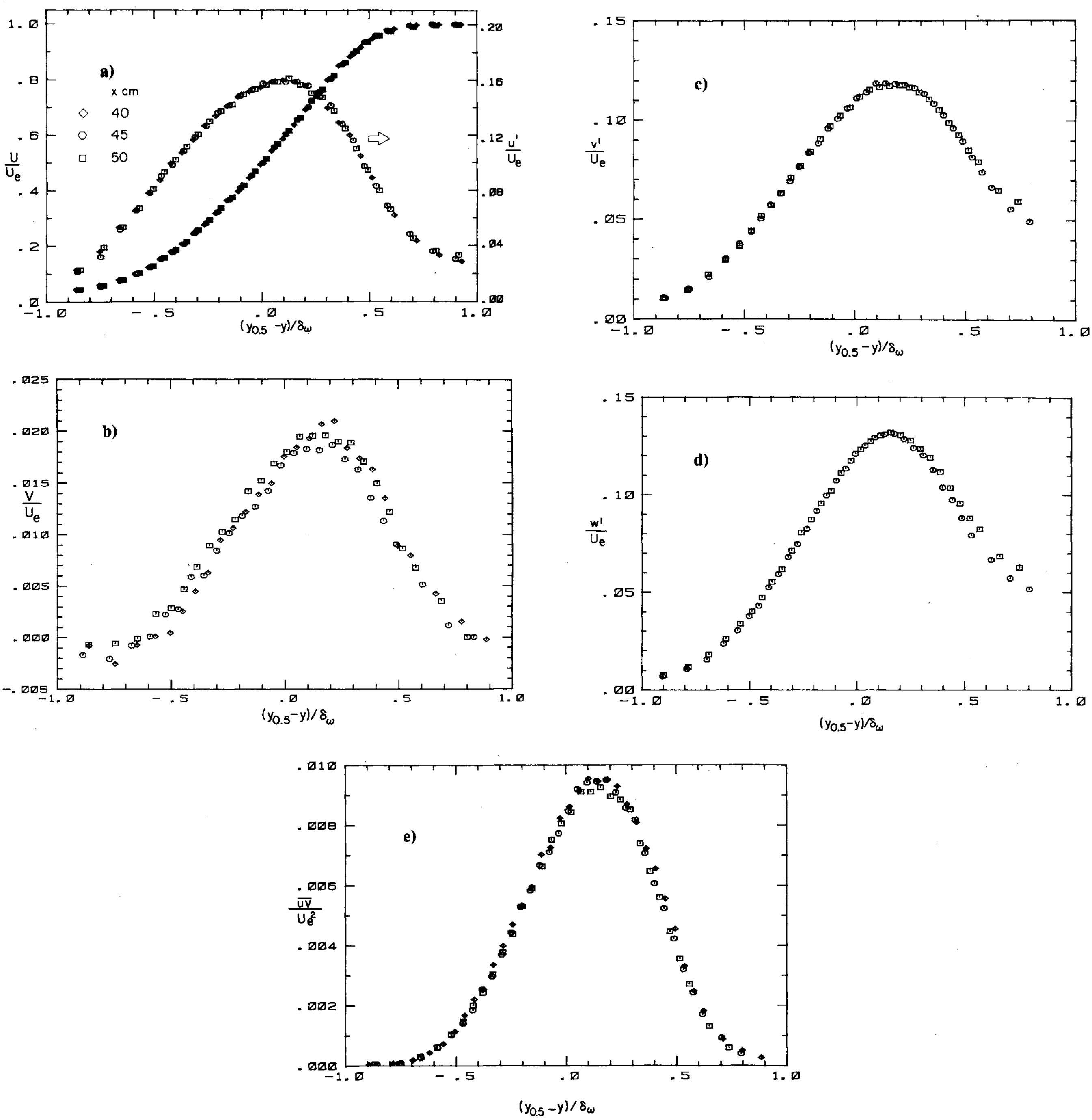
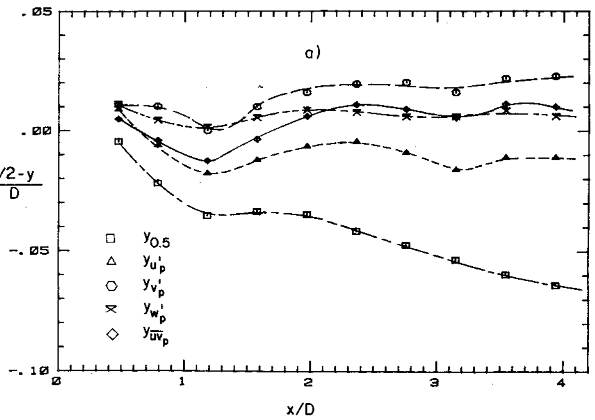


Fig. 5 Self-preserving profiles for the initially turbulent case: a) $U(y)$ and $u'(y)$; b) $V(y)$; c) $v'(y)$; d) $w'(y)$; e) $\overline{uv}(y)$.

Thus, it is clear that explanations for the separations between $y_{u'_p}$, $y_{v'_p}$, $y_{w'_p}$, $y_{\overline{uv}_p}$, and $y_{0.5}$ can be found through the large-scale coherent structures. However, because of the large dispersion in the shape, size, orientation, strength, formation time, and convection velocity of the naturally occurring structures, eduction of these structures has not been successful so far.[14,15,32,33] (Eduction through new processing techniques of the naturally occurring structures in the near- and far-fields of circular and plane jets and a plane mixing layer is being pursued in our laboratory.) Since the preferred mode of a jet is the most dominant and frequently occuring structure in the mixing layer of an unperturbed jet,[18,34] it will suffice if these separations could be explained from the results on "preferred mode" coherent structures. These structures in the axisymmetric mixing layer have been studied recently for different Reynolds numbers and initial conditions.[20] Details of these coherent structures – enhanced by controlled perturbation which allowed phase-locked eduction of the spatial distributions of properties without using the Taylor hypothesis – suggest that the relative transverse locations $y_{u'_p}$, $y_{v'_p}$, $y_{\overline{uv}_p}$, and $y_{0.5}$ are to be expected.[20] No inference regarding $y_{w'_p}$ can be made because azimuthal velocity was not measured in Ref. 20. The contours of phase-average vorticity show that the structure center is located outside the $y=D/2$ line, roughly at $y=y_{0.5}$ as shown in Fig 6. The peaks of phase-average transverse velocity intensity and Reynolds stress occur at smaller radii than that for the peak of longitudinal intensity. However, since time-average measures include contributions from both phase-random and coherent motions, the peaks of the coherent motion contours should also be taken into consideration. On this basis, the measured transverse locations $y_{u'_p}$, $y_{v'_p}$, $y_{\overline{uv}_p}$, and $y_{0.5}$ are quite consistent with the coherent structure data.

The larger separation among $y_{0.5}$, $y_{u'_p}$, and $y_{v'_p}$ for the



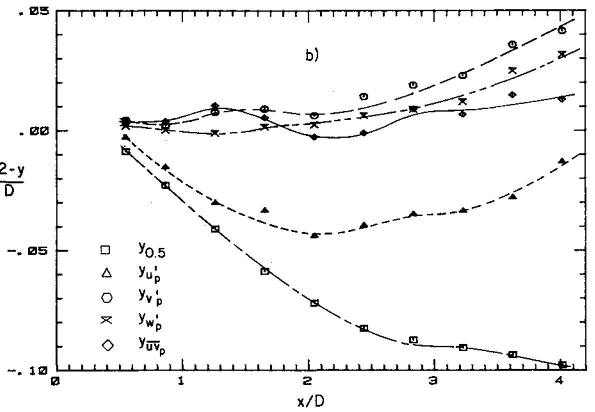


Fig. 6 Locations of the peaks of turbulence intensities and the Reynolds stress compared with $y_{0.5}$: a) laminar; b) turbulent.

initially turbulent case as compared to the initially laminar case, when compared with the detailed coherent structure data in Ref. 20, suggests that the coherent structure is more prevalent for the initially turbulent case; this claim was made earlier by the authors on the basis of high-speed flow-visualization movies of an axisymmetric mixing layer.[32] This is also consistent with the measured v'_{PT}, w'_{PT}, $\overline{uv}_{PT}$ being larger than v'_{PL}, w'_{PL}, $\overline{uv}_{PL}$, respectively.

Concluding Remarks

The axisymmetric mixing layer in the near-field of a circular jet is relevant to many applications involving mixing and noise production/control. Yet, data in this flow are extremely limited, and no detailed data on the normal and azimuthal intensities and Reynolds stress in this region could be found in the literature. This paper adds X-wire data to our earlier single wire data in the near-field mixing layer of a 12.7-cm air jet, and thus completes the documentation of the time-average characteristics of this flow.

In Ref. 25, the authors showed that the presence of a large exit plate has insignificant effect on the mixing layer integral measures, which, however, were found to be strongly dependent on the initial condition. It has been shown that an initially turbulent boundary layer produces a mixing layer with higher spread rate and higher peak values of u', v', w', and $\overline{uv}$ than the corresponding mixing layer with the initially laminar state. An attempt has been made to explain these time-average data on the basis of coherent structure data reported by us separately. These suggest that the effects of the initial condition continue to persist until the end of the potential core for the 12.7-cm jet. Clearly, the question of independence of the asymptotic state from the initial condition of an axisymmetric mixing layer cannot be resolved without investigating a larger jet; this question is also being pursued in a large plane mixing layer in our laboratory.

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